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SIMULTANEOUS DETERMINANTS OF MARKET SHARE AND ADVERTISING EXPENDITURE UNDER DYNAMIC CONDITIONS: THE CASE OF A FIRM WITHIN THE JAPANESE PHARMACEUTICAL INDUSTRY.

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I. INTRODUCTION

The major objective of this model is to analyze the simultaneous determination of market share and advertising expenditure under dynamic conditions. Obviously the relationship between market share and advertising expenditure depends upon the market structure of a particular industry, and thus it is essential that the model should be formulated so as to grasp the behavior of the decision makers, i.e. consumers and firms, within the industry concerned. Given the industrial framework, imperfect competitors within the industry would share many behavioral characteristics, with some degree of difference allowed for each company. Our empirical analysis is applied to the case of the Eisai Company within the Japanese pharmaceutical industry from 1959 to 1969. Although the model is limited to this particular company and industry, the theoretical formulations and estimation techniques together with the particular distributed lag used may be usefully applied to other cases.

There are many studies measuring and analyzing, in varying degrees, the cumulative effects of advertising on sales. For example, Nerlove and Waugh [13] have attempted to measure these effects in the case of oranges.(1) Palda [14] formulated the problem using the Koyck lag distribution and applied it to the case of the Lydia E. Pinkham Medicine Company. Telser [18] studied advertising and cigarettes.(2)

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* The authors would like to thank Professor K. Sato for his valuable comments on an earlier draft of the paper. This study comes out of an econometric model of the firm [23].

(1) In their empirical study Nerlove and Waugh adopted a log-linear demand equation in which the current advertising expenditure and the simple arithmatic average of advertising expenditure of the past 10 periods were used as the explanatory variables together with income and price variables.

(2) Telser's model implies in essence a transformed Koyck lag.

These studies are single equation analyses dealing with the causal flow of advertising expenditure to sales, but it is also desirable to analyze the causal flow of sales success and gains in market shares to advertising expenditure. Hence, in this study we will formulate market share equations as well as advertising expenditure equations, indicating their simultaneous relationship.

Advertising outlay may very well be regarded as a form of investment or capital formation. Using this concept, Nerlove and Arrow [12] provided a dynamic solution to the problem of establishing an optimal advertising policy for firms which are present value profit maximizers. In this paper, their solution is modified to the case of a firm which is a present value sales maximizer subject to present value minimum profit constraint.

Once the optimal advertising policy is given, and the advertising expenditure equation is formulated, the depreciation rate of current and past advertisement, or what Nerlove and Arrow call goodwill, will be estimated.

In the earlier investigations of Palda and Telser on the effects of advertising expenditure on sales, the Koyck lag was assumed *a priori*. To determine whether this particular lag formulation is indeed valid or not for our empirical case study, we will employ gamma distributed lags as proposed in [21] which can treat the Koyck lag as a special case.

In short, using a case study of a firm, the present paper proposes to do the following: (i) analyze the simultaneous determination of advertising expenditure and market share; (ii) formulate an optimal advertising policy for a firm whose behavioral goal is to maximize present value of sales subject to the minimum level of present valued profit; (iii) estimate the depreciation rate of goodwill, and (iv) check on the validity of the Koyck lag formulation using gamma distributed lags. Once the simultaneous model is estimated, then we will analyze the effects of the competitors' reaction (through an increase in their advertising expenditures) on the market share of the firm by making simulation exercises.

The organization of the paper is now in order: Section II presents behavioral hypotheses relating to industrial demand, market share, and advertising expenditure. As background knowledge for the reader, a brief discussion of the Japanese pharmaceutical industry and its market structure is given. Section III presents the estimated model, and its results are evaluated in Section IV. Finally, in Section V, the competitors' reaction to the firm's advertising policy is examined through some simulation exercises.

II. BEHAVIORAL HYPOTHESES OF INDUSTRIAL DEMAND, MARKET SHARE AND ADVERTISING EXPENDITURE

2.1 Japanese Pharmaceutical Industry and Its Market Structure: The Japanese pharmaceutical industry has grown in terms of output 4.8 times in the ten years between 1959 and 1968. Approximately 60 percent of the industrial output is produced by eleven companies, although the number of firms in the industry is said to be around 2,300. The combined market share of these eleven companies in ethical drugs[(3)] is estimated to reach 80 percent. Table 1 below presents the net sales of the top nine companies during the latter half of the 1968 fiscal period. The table indicates three groups of companies among the top nine firms: the first group is Takeda alone whose sales are more than three times as large as those of Sankyo, the second largest. The second group consists of three companies whose sales range between 20 and 24 billion yen, and the third group consists of five companies whose sales are 11 to 14.5 billion yen. The Eisai Company whose case we are analyzing belongs to the third group. However, the company is growing fast at the rate of 111 percent over four years between 1964 and 1968.

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(3) The pharmaceutical products are conventionally divided into two categories: "over-the-counter products" are those sold in drug stores, "ethical products" are those sold to hospitals.

Table 1

Net Sales and Growth Rates of Top Nine Companies

Company	Net Sales (100 million yen)	Growth Rate (1968/1964)
Group I		
Takeda	780	45.5%
Group II		
Sankyo	240	43.0
Shionogi	220	33.5
Tanabe	206	29.5
Group III		
Fujisawa	145	75.1
Eisai	130	111.0
Yamanouchi	127	81.0
Banyu	120	119.3
Daiichi	112	36.9

Source: Nihon Keizai Shinbun, June 20, 1969

One of the major reasons why the industry has grown so rapidly up to the present is that many companies have obtained licenses or know-how for manufacturing new products from foreign companies. This fact is illustrated in Table 2 which presents research and development (R&D) expenditure, the number of "star" products whose sales values are over 100 million yen per month, and the number of "star" products which are foreign license-based among the leading nine companies.

Except for the Eisai Company, all other companies have fairly high proportions of foreign licensed products among their "star" products. The stimulant of foreign licensing is expected to dwindle rapidly in the near future due to changes in strategies of foreign pharmaceutical companies. With the expected 'liberalization of capital' (as it is phrased in Japan), many foreign companies are already "landing" in Japan either to open their own subsidiaries or to buy up existing Japanese firms, rather than continuing the past strategy of selling "know-how" to the Japanese companies. If the Japanese companies are to survive or to compete with foreign subsidiaries, they will have to develop their own products. Consequently, the R&D expenditure may become a key factor, together with advertising expenditure, in determining the future of any firm.

Table 2

Research and Development (R&D) Expenditure and
Star Products of the Top Nine Companies

Firm	R&D Expenditure, (100 million yen) (1)	R&D Expenditure/Sales Ratio (%) (2)	Star Product Selling 100 million yen per month (3)	Star Product under foreign license (4)	(4)/(3) x 100 (%) (5)
Takeda	37.0	5.2	27	8	29.6
Sankyo	17.5	7.7	10	2	20.0
Shionogi	15.9	7.9	11	8	72.7
Tanabe	15.5	7.8	11	4	36.3
Fujisawa	12.7	9.3	10	7	70.0
Eisai	15.1	12.2	4	0	0.0
Yamanouchi	9.0	7.7	5	3	60.0
Banyu	2.3	2.0	6	4	66.7
Daiichi	9.7	9.3	8	3	37.5

Source: Nohon Keizai Shinbun, June 26, 1969

2.2 Consumer Demand for Pharmaceutical Products: The demand for pharmaceutical products is based on two propositions: the additive utility functions and the Gompertz curves.

The proposal of additive utility functions by Frisch (1959) and Houthakker (1960) has been quickly adopted to a number of empirical applications, since one can easily derive explicit consumer demand functions from it. From the maximization of an additive utility function

$$(2\text{-}1) \qquad u(x_1,\ldots,x_n) = u_1(x_1) + \ldots + u_n(x_n)$$

subject to budget constraint, Sato [16] derived a system of demand functions of the form

$$(2\text{-}2) \qquad d \ln x_i = \eta_i \, d \ln Y/P_a - \sigma\eta_i \, d \ln P_i/P_m \quad , i=1,\ldots,n$$

where x_i = the quantity of good i demanded by a consumer

Y = the money income of the consumer

P_a = the price index with average share weights, i.e.

$d \ln P_a = \Sigma\theta_i d \ln P_i \qquad \theta_i = P_i X_i/Y$

P_m = the price index with marginal share weights, i.e.

$$d \ln P_m = \Sigma \mu_i d \ln P_i, \quad \mu_i = P_i(\partial x_i/\partial Y)$$

η_i = the income elasticity of demand for good i

σ = overall average elasticity of substitution ($1/\sigma$ is equal to the income elasticity of the marginal utility of income).

(2-2) suggest double-log demand functions(4)

$$(2\text{-}3) \qquad \ln x_i = \eta_i \ln Y/P_a - \sigma\eta_i \ln P_i/P_m + \acute{\gamma}_i \text{ , } i=1,\ldots,n$$

and we shall use (2-3) in estimating the demand functions for pharmaceutical products.

As we noted in Section 2.1, the Japanese pharmaceutical industry is characterized by fast growth. When industry is growing fast, a major concern is to identify when the pace of fast growth will slow down. A mathematical function to express the life cycle of such growth may take the form of a logistic curve. The usual logistic curve is given by

$$(2\text{-}4) \qquad x_i = \frac{x_i^*}{1 + me^{-\alpha t}}$$

while the Gompertz curve is given by

$$(2\text{-}5) \qquad x_i = x_i^* a^{b^t} \quad , \quad 0 < b < 1$$

where x_i^* is the expected demand for commodity i. The differentiation of (2-4) and (2-5) respectively yield

$$(2\text{-}6) \qquad \frac{dx_i}{dt} = \alpha\, x_i\left(1 - \frac{x_i}{x_i^*} \right)$$

$$(2\text{-}7) \qquad \frac{d \ln x_i}{dt} = \frac{1}{x_i} \frac{dx_i}{dt} = - \ln b(\ln x_i^* - x_i) = \beta(\ln x_i^* - x_i)$$

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(4) In order to integrate (2-2) to derive (2-3), one has to assume constant coefficients, and as Sato [16] says this is indeed a stringent assumption, but (2-3) is usually adopted in empirical studies as an approximation.

where $\beta = -\ln b$ and it may be called a reaction coefficient.

The usual logistic curve (2-4) has a property that it is symmetric around the point of inflection, while the Gompertz curve is not. Which curve should be adopted is an empirical question. After preliminary examination we found that our data tend to support the Gompertz curve.(5) Then following Chow [2] (2-7) may be approximated in discrete form

$$\text{(2-8)} \qquad \ln x_{i,t} - \ln x_{i,t-1} = \beta_i(\ln x^*_{i,t} - \ln x_{i,t-1})$$

or

$$\text{(2-8)'} \qquad \ln x_{i,t} = \beta_i \sum_{k=0}^{m_i-1} (1-\beta_i)^k \ln x^*_{i,t-k} + (1-\beta_i)^{m_i} \ln x_{i,t-m_i}$$

$$= \beta_i \sum_{k=0}^{m_i-1} (1-\beta_i)^k L^k \ln x^*_{i,t} + (1-\beta_i)^{m_i} \ln x_{i,t-m_i}$$

where $x_{i,t-m_i}$ is the initial value of x_i, and L is the distributed lag operator (or the backward shift operator)(6), and as $m_i \to \infty$, we will have

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(5) (2-6) and (2-7) may be approximated in discrete form by putting $dx_i = \Delta x_{i,t}$ and $dt = \Delta t$:

$$\text{(2-6)'} \qquad \frac{\Delta x_{i,t}}{x_{it}} = \alpha - \frac{\alpha}{x^*} x_{i,t-1}$$

and $\quad \text{(2-7)'} \quad \Delta \ln x_{i,t} = \beta_i \ln x^*_{i,t} - \beta_i \ln x_{i,t-1}$

Then given data we may plot $\Delta x_{i,t}/x_{i,t}$ against $x_{i,t-1}$ and $\Delta \ln x_{i,t}$ against $\ln x_{i,t-1}$ and see which may fit better into a linear function. Our data suggest the choice of the Gompertz curve over the logistic curve.

(6) The word distributed lag operator is the term used in econometrics, whereas backward shift operator is used in electrical engineering and statistical time series analysis. This operator forms an algebraic space of a linear operator and in the frequency domain (i.e. Fourier transform) we have $L \doteq \exp(-i2\pi f\Delta)$, where Δ is the time interval. An interested reader should consult [1] or [3] . B(L) in (2-9) is called the transfer function or, in its Fourier transform, generalized frequency function.

$$(2\text{-}9) \qquad \ln x_{i,t} = \frac{\beta_i}{1-(1-\beta_i)L} \ln x^*_{i,t} = \frac{\Phi(L)}{\Psi(L)} \ln x^*_{i,t} = B(L) \ln x^*_{i,t}$$

provided that $|\beta_i| < 1$ and $|(1-\beta_i)L| < 1$, and $\Phi(L) = \beta_i$, $\Psi(L) = 1-(1-\beta_i)L$.

The transform function B(L) in (2-9) is the Koyck lag with the moving average operator $\Phi(L)= \beta_i$, and the autoregressive operator $\Psi(L)=1-(1-\beta_i)L$. In many empirical studies the Koyck lag is estimated by premultiplying (2-9) by the autoregressive operator $\Psi(L)$ and then rearranging terms. However, it is well known that this transformation generates an autocorrelated disturbance term and the ordinary least squares (or for that matter any simultaneous estimation procedure which does not take care of autocorrelated error terms) estimates will generally be inconsistent.

Faced with this problem, we may directly estimate B(L) from (2-8)'. If m_i is sufficiently large, the last term of (2-8)' may be treated as constant. Then our basic demand function to be estimated is given by

$$(2\text{-}10) \qquad \ln x_{i,t} = B(L) \ln x^*_{i,t} + \gamma_{o,i} + u_{i,t}$$

where $B(L) = \sum_{k=0}^{m_i-1} \beta_{i,k}L^k$, with $\beta_{i,k} = \beta_i(1-\beta_i)^k$, $\gamma_{o,i} = (1-\beta_i)^{m_i} \ln x_{i,t-m_i}$,

and $u_{i,t}$ is the disturbance term.

The coefficients of L^k in the polynomial B(L) in (2-10), $\beta_{i,k}$, will be geometrically declining as the power k increases. One empirical question one might ask is whether the given data will indeed imply the Koyck lag. To answer this question B(L) in (2-10) is represented in more general form by the gamma distributed lags proposed in [21], $\{\beta_{i,k} = k^{s_i-1} e^{-k}\}$, where s_i is an unknown parameter to be estimated, and for this purpose we use the nonlinear least squares estimation technique [11]. (7) If preliminary investigation indicates that the coefficient of L^k indeed declines as the

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(7) The consistency of the nonlinear least squares estimator under certain regularity conditions is proven by Hartley and Booker [8] . To estimate the gamma distributed lags it is necessary to give a priori the value of m_i, i.e. the length of lags. We allowed as great a number of lags as possible with our data. This point is discussed in Section IV.

Koyck lag indicates, then the gamma distributed lags, $\{k^{s_i-1}e^{-k}\}$ are replaced by $\{e^{-\alpha_i k}\}$ which can be shown to be the Koyck lag, and the equation is re-estimated with this particular form of the gamma distributed lag.

The Gompertz curve formulation (2-8) may be interpreted as the usual adaptive adjustment process in log-linear form, and we may represent the expected demand, $x^*_{i,t}$ by (2-3). Then from (2-3) and (2-10) we will obtain

$$(2\text{-}10)' \quad \ln x_{i,t} = \eta_i B(L)(\ln Y_t/P_{a,t} - \sigma \ln P_{i,t}/P_{m,t}) + \gamma'_{i,o} + u_{i,t}$$

where $\gamma'_{i,o}$ is the new constant term. Since the price index with marginal weighting, P_m is virtually impossible to obtain, we will represent P_m by the price index with average weighting, P_a. Rather than estimating the over-all average elasticity of substitution σ from (2-10)', we will use the estimate of the Japanese Economic Planning Agency given in Sato [16] , which is $\hat{\sigma} = .687$.(8)

The demand equation (2-10)' is estimated for the following eight product groups on a per capita basis: (i) agents affecting the nervous system and sensory organs; (ii) cardiovascular agents and agents affecting respiratory organs; (iii) agents affecting digestive organs and agents for uro-genital and anal organs; (iv) vitamin preparations: (v) nutrients, tonics and alternatives; (vi) dermatological agents; (vii) chemotherapeutics and antibiotic preparations and (viii) miscellaneous pharmaceutical products. This classification conforms to the practice in the industry. For the income variable, $Y_t/P_{a,t}$, we shall use per capita real personal consumption and medicare expenditure. P_a is represented by the price index of consumer expenditure.

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(8) σ can be estimated directly from (2-11). However, our data indicate a strong multicollinearity between $Y_t/P_{a,t}$ and $P_{i,t}/P_{a,t}$, thus making the estimation difficult. This is due to the fact that while the real expenditure of the consumer has grown steadily, the price index of pharmaceutical product i, $P_{i,t}$, relative to the price index of consumer expenditure, $P_{a,t}$ has tended to decline.

2.3 Market Share Determination: Whether a firm tries to expand its market share or not depends upon the firm's behavioral goal. One of the main objectives of the Eisai Company is present valued sales maximization subject to a minimum level of the present value of profit.

If the demand for the industry as a whole is given, then sales maximization may be interpreted as maximization of the market share, provided that the firm is a price taker.[9] Among the variables the Company controls in order to increase its market share are advertising expenditure and research and development (R & D) expenditure. The relationship between the R & D expenditure and the development of new products is now receiving more attention and so it should. As much as we would like to explore this further, the available data on R & D activities of the firm we investigated are too rough to enable thorough analysis. Consequently, we will focus here on the relationship between market share and advertising expenditure. For this purpose, we formulate a simple hypothesis that Eisai's expected share of total industrial demand is proportional to its share of advertising expenditure:

$$\text{(2-11)} \quad \ln (E^e_{i,t}) = \mu_i \ln(\frac{AE_t}{AO_t} X_{i,t}) \text{ [10]}$$

and we introduce the usual adaptive adjustment process:

$$\text{(2-12)} \quad \ln E_{i,t} - \ln E_{i,t-1} = \theta_i(\ln E^e_{i,t} - \ln E_{i,t-1}) \text{ [11]}$$

where $X_{i,t}$ = industrial demand for the i-th product at time t

$E^e_{i,t}$ = expected demand for the i-th product of Eisai at time t

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(9) How prices of pharmaceutical products are determined will be discussed at the end of Section IV.

(10) The hypothesis expressed in (2-11) is not as simple as it may look. Many Japanese pharmaceutical firms regard sales and advertising promotion activities as the only means of increasing their market shares and sales. Eisai is quite sensitive to the sales and advertising promotions of its competitors and it makes an effort to keep up with (or overtake) its rivals in these activities.

One may wonder whether the two variables, AE_t and AO_t in (2-11) should be replaced by the advertising expenditure for a particular product, $AE_{i,t}$ and $AO_{i,t}$. We have information on $AE_{i,t}$ and $AO_{i,t}$. We have information on $AE_{i,t}$ but not on $AO_{i,t}$. Furthermore, advertising expenditure by the firm tends to have a spill-over effect in the sense that even if the advertisement is on product i, it may serve to promote any other product of the Company.

(11) (2-12) may imply the Gompertz curve formulation as in (2-8). The reason why we chose the logarithmic adjustment process is because this formulation implies constant elasticity of demand with respect to advertising expenditure and the constant elasticity is assumed in deriving the optimal advertising policy. See Section 2.4.

$E_{i,t}$ = actual demand for the i-th product of Eisai at time t

AE_t = net stock of effects of current and past advertising expenditure, or goodwill, of Eisai at time t

AO_t = net stock of effects of current and past advertising expenditure, or goodwill, of companies other than Eisai at time t.

From (2-11) and (2-12) we will obtain

$$\text{(2-13)} \qquad \ln E_{i,t} = \theta_i \sum_{k=0}^{n_i-1} (1-\theta_i)^k \ln \frac{AE_{t-k}}{AO_{t-k}} X_{i,t-k} + (1-\theta_i)^{n_i} \ln E_{i,t-n_i} + \gamma_i$$

where γ_i is a constant term containing $(1-\theta_i)$ and μ_i. If n_i is sufficiently large, or if $(1-\theta_i)$ is sufficiently small, then treating the term, $(1-\theta_i)^{n_i} \ln E_{i,t-n_i}$ as zero may not incur much error. (2-13) indicates again the Koyck lag. Here, however, we will examine whether the Koyck lag coefficients are indeed valid or not by adopting the same procedure as we discussed in the estimation of the industrial demand functions; thus the equation to be estimated becomes

$$\text{(2-13)'} \qquad \ln E_{i,t} = C(L) \ln \frac{AE_t}{AO_t} X_{i,t} + \gamma_i + e_{it}$$

where e_{it} is the disturbance term, and the transform function $C(L)$ is given by the gamma distributed lags:

$$C(L) = \sum_{k=1}^{n_i+1} k^{s_i-1} e^{-k} L^{k-1} \quad \text{or} \quad C(L) = \sum_{k=1}^{n_i+1} e^{-\alpha_i k} L^{k-1}$$

(2-13)' will be estimated for the following six product groups: (i) agents affecting the nervous system and sensory organs; (ii) cardiovascular agents and agents affecting respiratory organs; (iii) agents affecting digestive organs and agents for uro-genital and anal organs; (iv) vitamin preparations; (v) nutrients, tonics and alternatives; (vi) dermatological agents. We will treat the two remaining product groups as exogenous variables, since the Eisai Company just began producing chemotherapeutics and antibiotics (group vii) in the second half of 1967, and since miscellaneous products (group viii) represent at most 3% of the total sales of the company. For the estimation of (2-13)' we will use the nonlinear two-stage least squares used in [22].

2.4 Advertising Expenditure: In Section 2.3 we indicated that the Eisai Company's goal is to maximize the present value of sales subject to a minimum level of the present valued profit. Under this proposition let us derive an optimal advertising policy as follows. First we start with the following three assumptions.

(1) The firm's net stock of effects of current and past advertising expenditure, goodwill, obeys the formula

$$(2\text{-}14) \qquad \dot{A} + \delta A = a, \quad \text{or} \quad g = \dot{A} + \delta A - a = 0$$

where $\dot{A} = \frac{dA}{dt}$, a is current advertising expenditure and δ is the depreciation rate of goodwill.

(2) The firm's market share function is given by

$$(2\text{-}15) \qquad m = m(A, z)$$

where z is any other variable (it can be a vector) which the firm does not control, e.g. AO. The industrial demand, Q, is given by the consumer's choice:

$$(2\text{-}16) \qquad Q = Q(Y_d/P_a,\ P/P_a)$$

where Y_d/P_a, is per capita real income and P_a is the price index of consumer expenditure. Then the firm's demand q is

$$(2\text{-}17) \qquad q = mQ = q(A, P, z_1)$$

where z_1 is again a variable exogenous to the firm and includes P_a.

(3) Given the cost function

$$(2\text{-}18) \qquad C = C(q)$$

the profit, Π, is given by

$$\Pi = Pq - C(q) - a = Pq(A, z_1) - C(q) - a = R(P, A, z_1) - a$$

where P is the price of q. Then the minimum level of the present valued profit to be maintained, Π^*, may be given by

$$\text{(2-19)} \qquad \Pi^* = \int_0^\infty e^{-rt}\{P\cdot q(A, P, z_1) - C(q) - a\}\, dt = \int_0^\infty \Psi\, dt$$

where r is the discount rate.

Our problem is now stated in the following Lemma.

LEMMA 1: Under the assumptions (1), (2), and (3) above and given the initial value $A(0)=A_o$, the time path of A which maximizes

$$\text{(2-20)} \qquad \int_0^\infty e^{-rt} P\cdot q(A, P, z_1)dt = \int_0^\infty \Phi\, dt$$

subject to the constraints (2-14) and (2-19) is given by

$$A = \frac{\omega}{(\S-1)(\delta+r)} C'q$$

where $\S = -\frac{\partial q}{\partial P}\frac{P}{q}$ $\omega = \frac{\partial q}{\partial A}\frac{A}{q}$; and $C' = \frac{\partial C}{\partial q}$.

PROOF: This is an isoperimetric problem. Let

$$H(A, P, \mu, \lambda) = \int_0^\infty \{\Phi - \mu\Psi - \lambda(t)g\}\, dt = \int_0^\infty F\, dt$$

Then the necessary conditions for a maximum are given by the following Euler equations:

$$\text{(2-20)} \qquad F_A - \frac{d}{dt}F_{\dot{A}} = 0$$

$$\text{(2-21)} \qquad F_a - \frac{d}{dt}F_{\dot{a}} = 0$$

$$\text{(2-22)} \qquad F_P - \frac{d}{dt}F_{\dot{P}} = 0.$$

(2-20), (2-21) and (2-22) become

$$(2\text{-}23)\qquad F_A - \frac{d}{dt}F_{\dot{A}} = e^{-rt}P\,\frac{\partial q}{\partial A} - \mu e^{-rt}\left(P - \frac{\partial C}{\partial q}\right)\frac{\partial q}{\partial A} - \delta\lambda(t) + \frac{d}{dt}\lambda(t) = 0$$

$$(2\text{-}24)\qquad F_a - \frac{d}{dt}F_{\dot{a}} = \mu e^{-rt} + \lambda(t) = 0$$

$$(2\text{-}25)\qquad F_P - \frac{d}{dt}F_{\dot{P}} = e^{-rt}\left\{q + P\,\frac{\partial q}{\partial P} - \mu\left(q + P\,\frac{\partial q}{\partial P} - \frac{\partial C}{\partial q}\frac{\partial q}{\partial P}\right)\right\} = 0.$$

From (2-24) we obtain

$$(2\text{-}26)\qquad \lambda(t) = -\mu e^{-rt}, \text{ and } \quad \frac{d}{dt}\lambda(t) = r\,e^{-rt}.$$

Substituting (2-26) into (2-23) and rearranging the terms we have

$$P\,\frac{\partial q}{\partial A} - \mu\left(P - \frac{\partial C}{\partial q}\right)\frac{\partial q}{\partial A} + \delta\mu + r\mu = 0$$

or

$$(2\text{-}27)\qquad \mu = \frac{P\,\frac{\partial q}{\partial A}}{(P - C')\frac{\partial q}{\partial A} - (\delta + r)}$$

where $C' = \frac{\partial C}{\partial q}$. On the other hand from (2-25) we obtain

$$(2\text{-}28)\qquad = \frac{q(1 - \S)}{q(1 - \S) - C'\,\frac{\partial q}{\partial P}}$$

where $\S = -\frac{\partial q}{\partial P}\frac{P}{q}$. Equating (2-27) to (2-28) and rearraning the terms we get

$$(2\text{-}29)\qquad A = \frac{\omega}{(\S - 1)(\delta + r)}\;C'q \;= k_1 C'q$$

where $\omega = \frac{\partial q}{\partial A}\frac{A}{q}$, and $k_1 = \frac{\omega}{(\S - 1)(\delta + r)}$.

(2-29) gives the optimal advertising policy. Whether or not the proportional factor k_1 stays constant will depend on whether ω, §, δ, and r are constant. Our demand function (2-2) implies a constant price elasticity and we have formulated (2-11) and (2-12) to conform to the constant elasticity with respect to goodwill, ω.

In actuality the optimal condition (2-29) may only be attained with some adjustment over time, and thus we apply Jorgenson's investment function [10] as follows. (2-14) indicates that current advertising expenditure, a, consists of net goodwill investment, $\dot{A}$, and replacement, δA. Now suppose the net goodwill investment in a discrete time period, NA_t, is given by

$$(2\text{-}30) \qquad NA_t = \tau(L)(A^*_t - A^*_{t-1})$$

where $\tau(L)$ is the transform function and A^*_t is the expected goodwill at time t. Then the current advertising expenditure, PRE_t, will be given by

$$(2\text{-}31) \qquad PRE_t = NA_t + \delta AE_{t-1} = \tau(L)\ (A^*_t - A^*_{t-1}) + \delta AE_{t-1}$$

and if we treat (2-29) as representing the expected net stock of goodwill, then (2-31) now becomes

$$(2\text{-}32) \qquad PRE_t = k_1\tau(L)\ (C'_t q_t - C'_{t-1}q_{t-1}) + \delta AE_{t-1}.$$

We do not have the marginal cost estimate, C'_t, so let us represent $C'_t q_t$ by the net sales value, V_t. This is equal to the assumption that the marginal cost is proportional to the price. AE_{t-1} in (2-32) is the net stock of goodwill, and given the identity

$$AE_t = NA_t + \delta AE_{t-1}$$

and (2-31), we will obtain

$$(2\text{-}33) \qquad AE_t = PRE_t + (1-\delta)AE_{t-1} = \sum_{k=0}^{\ell} (1-\delta)^k PRE_{t-k} + (1-\delta)^{\ell} AE_{t-\ell}.$$

If δ is sufficiently large or if ℓ is large, we may treat the second term as zero. Substituting (2-33) into (2-32) and adding the disturbance term ε_t, we obtain

$$(2\text{-}34) \qquad PRE_t = k_1\tau(L)\,(V_t - V_{t-1}) + \delta\Big(\sum_{k=0}^{\ell}(1-\delta)^k PRE_{t-k-1}\Big) + \varepsilon_t$$

It is better to have a large integer value for ℓ on the summation sign of the second term above, but this is constrained by the availability of data, and in our empirical estimation we have to set ℓ equal to 7. As in the demand and market share equations given earlier, let us represent the coefficients of L^k in the transform function $\tau(L)$ by the gamma distributed lag coefficients $\{k^{s-1}e^{-k}\}$. For estimating the market share equations (2-13) and the advertising expenditure (2-34) we shall use nonlinear two-stage least squares.(12)

III. THE ESTIMATED MODEL

On the basis of semi-annual data for twenty-one observations from the second half of 1959 to the second half of 1969,(13) the behavioral equations presented in the previous section are estimated by such methods as nonlinear least squares (NLLS) and nonlinear two-stage least squares (NL2SLS). $\bar{R}^2$ denotes the coefficient of determination adjusted for degrees of freedom, and DW indicates the Durbin-Watson test statistic. The figure just below a coefficient is the estimated standard error of the coefficient. $\frac{1}{G}$, or $\frac{1}{Z}$ in front of each gamma distributed lag is to make the sum of the lag coefficients equal to unity, e.g. $G = \Sigma e^{-ak}$, and $Z = \Sigma\, k^{s-1}e^{-k}$.

The List of Variables

The variables are given in alphabetical order. An endogenous variable in the system is indicated by an asterisk in the upper left hand corner.

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(12) The statistical properties of this procedure (e.g. consistency) are not known yet. This will be discussed in Section IV.

(13) This does not include the lagged observations necessary for the estimation of the distributed lags.
The Eisai Company has a half-a-year fiscal period of April-September and October-March. All the data were arranged to conform to this fiscal period classification.

$^*AE_t$ = net stock of <u>goodwill</u> of Eisai at time t, millions of 1965 yen

AO_t = net stock of <u>goodwill</u> of ten companies excluding Eisai at time t, millions of 1965 yen. These ten companies are, in alphabetical order, Banyu, Chugai, Daiichi, Dai-Nippon, Fujisawa, Takeda, Tanabe, Sankyo, Shionogi and Yamanouchi.

$\bar{E}_{BC,t}$ = Eisai's sales of chemotherapeutics and antibiotics at time t, millions of current yen

$^*E_{CR,t}$ = Eisai's sales of cardiovascular agents and agents affecting respiratory organs at time t, millions of 1965 yen

$^*E_{D,t}$ = Eisai's sales of agents affecting digestive organs and agents for uro-genital and anal organs at time t, millions of 1965 yen

$^*E_{DER,t}$ = Eisai's sales of dermatological agents at time t, millions of 1965 yen

$^*E_{NS,t}$ = Eisai's sales of agents affecting the nervous system and sensory organs at time t, millions of 1965 yen

$^*E_{NU,t}$ = Eisai's sales of nutrients, tonics and alternatives at time t, millions of 1965 yen

$\bar{E}_{OT,t}$ = Eisai's sales of miscellaneous products at time t, millions of current yen

$^*E_{V,t}$ = Eisai's sales of vitamin preparations at time t, millions of 1965 yen

N_t = total population at time t, thousands of persons

$P_{d,t}$ = price index of consumer expenditure at time t, 1965=100

$P_{i,t}$ = industrial price level of pharmaceutical product i at time t, 1965=100, i=NS,CR,D,V,NU,DER,BC and OT

$PE_{i,t}$ = price index of Eisai's i-th product at time t, 1965=100, i=NS, CR,D,V,NU, and DER

$^*PRE_t$ = Eisai's advertising expenditure at time t, millions of 1965 yen

$^*V_t$ = net sales of Eisai at time t, millions of current yen

$^*x_{BC,t}$ = per capita industrial production of chemotherapeutics and antibiotics at time t, thousands of 1965 yen

$^*x_{CR,t}$ = per capita industrial production of cardiovascular agents and agents affecting respiratory organs at time t, thousands of 1965 yen

$^*x_{D,t}$ = per capita industrial production of agents affecting digestive organs and agents for uro-genital and anal organs at time t, thousands of 1965 yen

$^*x_{DER,t}$ = per capita industrial production of dermatological agents at time t, thousands of 1965 yen

$^*x_{NS,t}$ = per capita industrial production of agents affecting the nervous system and sensory organs at time t, thousands of 1965 yen

$^*x_{NU,t}$ = per capita industrial production of nutrients, tonics, and alternatives at time t, thousands of 1965 yen

$^*x_{OT,t}$ = per capita industrial production of miscellaneous pharmaceutical products at time t, thousands of 1965 yen

$^*x_{V,t}$ = per capita industrial production of vitamin preparations at time t, thousands of 1965 yen

$^*X_{i,t}$ = industrial production of the i-th product at time t, millions of 1965 yen, i=NS, CR,D,V,NU,DER,BC, and OT

$^*y_t$ = per capita real personal consumption and medicare expenditure at time t, millions of 1965 yen.

(I) Industrial Demand Block

(3- 1)
$$\ln x_{NS,t} = \underset{(.0592)}{1.5036}\ \frac{1}{G}\sum_{k=1}^{6} e^{-ak}\left(\ln y_{t-k+1} - .687\ \ln\frac{P_{NS,t-k+1}}{P_{d,t-k+1}}\right) - \underset{(.5299)}{15.6587}$$

$$a = \underset{(.7082)}{2.0927}$$

$\bar{R}^2$ = .972

DW = 1.16

NLLS

(3- 2)
$$\ln x_{CR,t} = \underset{(.0759)}{2.0148}\ \frac{1}{Z}\sum_{k=1}^{15} k^{s-1}e^{-k}\left(\ln y_{t-k+1} - .687\ \ln\frac{P_{CR,t-k+1}}{P_{d,t-k+1}}\right) - \underset{(.5180)}{20.6476}$$

$$s = \underset{(1.1370)}{7.1389}$$

$\bar{R}^2$ = .991

DW = 1.72

NLLS

(3- 3)
$$\ln x_{DU,t} = \underset{(.0438)}{1.3918}\ \frac{1}{G}\sum_{k=1}^{14} e^{-ak}\left(\ln y_{t-k+1} - .687\ \ln\frac{P_{DU,t-k+1}}{P_{d,t-k+1}}\right) - \underset{(.3805)}{15.1490}$$

$$a = \underset{(.5768)}{.6635}$$

$\bar{R}^2$ = .983

DW = 1.50

NLLS

$$\text{(3-4)} \qquad \ln x_{V,t} = \underset{(.2210)}{1.1814} \frac{1}{G} \sum_{k=1}^{3} e^{-ak} \left(\ln y_{t-k+1} \underset{(.0118)}{-12.5928} \cdot .687 \ln \frac{P_{V,t-k+1}}{P_{d,t-k+1}} \right)$$

$a= 15.0290$
$(.1145)$

$\bar{R}^2 = .887$
DW = .221
NLLS

$$\text{(3-5)} \qquad \ln x_{NU,t} = \underset{(.1044)}{1.6688} \frac{1}{G} \sum_{k=1}^{3} e^{-ak} \left(\ln y_{t-k+1} - .687 \ln \frac{P_{NU,t-k+1}}{P_{d,t-k+1}} \right)$$

$a=14.4689$
$(.0883)$

-17.5674
$(.0059)$

$\bar{R}^2 = .987$
DW = .482
NLLS

$$\text{(3-6)} \qquad \ln x_{DER,t} = \underset{(.0658)}{.4815} \frac{1}{G} \sum_{k=1}^{3} e^{-ak} \left(\ln y_{t-k+1} - .687 \ln \frac{P_{DER,t-k+1}}{P_{d,t-k+1}} \right)$$

$a= 13.0273$
$(.0058)$

-6.5739
$(.0025)$

$\bar{R}^2 = .881$
DW = .582
NLLS

$$\text{(3-7)} \qquad \ln x_{BC,t} = \underset{(.0357)}{1.2775} \frac{1}{G} \sum_{k=1}^{6} e^{-ak} \left(\ln y_{t-k+1} - .687 \ln \frac{P_{BC,t-k+1}}{P_{d,t-k+1}} \right)$$

$a= 2.0750$
$(.8883)$

-13.4777
$(.3035)$

$\bar{R}^2 = .988$
DW = 1.19
NLLS

$$\text{(3-8)} \qquad \ln x_{OT,t} = \underset{(.1326)}{.5011} \frac{1}{Z} \sum_{k=1}^{12} k^{s-1} e^{-k} \left(\ln y_{t-k+1} - .687 \ln \frac{P_{OT,t-k+1}}{P_{d,t-k+1}} \right)$$

$s= 2.0757$
$(.7757)$

-6.8940
(1.1057)

$\bar{R}^2 = .472$
DW = .472
NLLS

(II) Eisai's Market Share Equations

(3-9) $$\ln E_{NS,t} = \underset{(.0795)}{.7717}\ \frac{1}{G}\sum_{k=1}^{6} e^{-ak} \ln\left(\frac{AE_{t-k+1}}{AO_{t-k+1}} X_{NS,t-k+1}\right) \underset{(.5860)}{+\ 1.1036}$$

$a = \underset{(1.0766)}{2.0474}$

$\bar{R}^2 = .927$
DW = 1.91
NL2SLS

(3-10) $$\ln E_{CR,t} = \underset{(.2453)}{.6584}\ \frac{1}{G}\sum_{k=1}^{3} e^{-ak} \ln\left(\frac{AE_{t-k+1}}{AO_{t-k+1}} X_{CR,t-k+1}\right) \underset{(.0222)}{+\ 2.3852}$$

$a = \underset{(.0684)}{7.8000}$

$\bar{R}^2 = .771$
DW = .31
NL2SLS

(3-11) $$\ln E_{DU,t} = \underset{(.2731)}{.4426}\ \frac{1}{Z}\sum_{k=1}^{7} k^{s-1}e^{-k} \ln\left(\frac{AE_{t-k+1}}{AO_{t-k+1}} X_{DU,t-k+1}\right) \underset{(.6463)}{+\ 4.3522}$$

$s = \underset{(1.6574)}{3.9282}$

$\bar{R}^2 = .590$
DW = .69
NL2SLS

(3-12) $$\ln E_{V,t} = \underset{(.1282)}{1.4743}\ \frac{1}{G}\sum_{k=1}^{7} e^{-ak} \ln\left(\frac{AE_{t-k+1}}{AO_{t-k+1}} X_{V,t-k+1}\right) \underset{(1.1929)}{-\ 4.7774}$$

$a = \underset{(.4652)}{.7715}$

$\bar{R}^2 = .918$
DW = 1.31
NL2SLS

(3-13) $$\ln E_{NU,t} = \underset{(.2109)}{1.5202}\ \frac{1}{Z}\sum_{k=1}^{7} k^{s-1}e^{-k} \ln\left(\frac{AE_{t-k+1}}{AO_{t-k+1}} X_{NU,t-k+1}\right) \underset{(3.2992)}{-\ 5.2426}$$

$s = \underset{(1.8833)}{3.6219}$

$\bar{R}^2 = .803$
DW = .52
NL2SLS

(3-14) $$\ln E_{DER,t} = \underset{(.3833)}{1.5510}\ \frac{1}{Z}\sum_{k=1}^{7} k^{s-1}e^{-k}\ \ln\left(\frac{AE_{t-k+1}}{AO_{t-k+1}}X_{DER,t-k+1}\right) - \underset{(2.4934)}{4.7287}$$

$$s = \underset{(1.1056)}{3.4408}$$

$\bar{R}^2 = .548$
DW = 2.95
NL2SLS

(III) Advertising Expenditure

(3-15) $$PRE_t = \underset{(.1557)}{.5902}\ \frac{1}{Z}\sum_{k=1}^{7} k^{s-1}e^{-k}(V_{t-k+1} - V_{t-k}) + \underset{(.0328)}{.4494}\ AE_{t-1}$$

$$s = \underset{(1.4683)}{2.7009}$$

$\bar{R}^2 = .988$
DW = 2.01
NL2SLS

(IV) Identities

(3-16) $$X_{i,t} = x_{i,t}N_t \qquad i=NS,CR,DU,V,NU,DER,BC,\text{ and }OT$$

(3-17) $$V_t = \sum_i PE_{i,t}E_{i,t} + \bar{E}_{BC,t} + \bar{E}_{OT,t}$$

$$i=NS,CR,DU,V,NU,\text{ and }DER$$

(3-18) $$AE_t = \sum_{h=0}^{7} (.5506)^h\ PRE_{t-h}$$

IV. EVALUATION OF THE BEHAVIORAL EQUATIONS

4.1 Industrial Demand Equations: As discussed in Section 2.2, consumer demand for pharmaceutical products was based on the additive utility function and the Gompertz curve. Interesting empirical results associated with these propositions may be the following points: (i) income elasticities, η_i ;

(ii) price elasticities, $\sigma\eta_i$ (14) and (iii) time response form of distributed lags. These results are in the estimated equations from (3-1) to (3-8), and tabulated in Tables 3 and 4 below.

Table 3

Estimates of Income Elasticities ($\hat{\eta}_i$) and "Own-price" Elasticities ($\hat{\sigma}\hat{\eta}_i$) of Japanese Pharmaceutical Products

Per capita demand product groups	Income elasticities($\hat{\eta}_i$)	"Own-price" elasticities($\hat{\sigma}\hat{\eta}_i$) with $\hat{\sigma}$= .687
1. agents affecting the nervous system and sensory organs, x_{NS}	1.50	-1.03
2. cardiovascular agents and agents affecting respiratory organs, x_{CR}	2.01	-1.38
3. agents affecting digestive organs and agents for uro-genital and anal organs, x_{DU}	1.39	-.95
4. vitamin preparations, x_V	1.18	-.81
5. nutrients, tonics and alternatives, x_{NU}	1.67	-1.15
6. dermatological agents, x_{DER}	.48	-.33
7. chemotherapeutics and anti-biotic preparations, x_{BC}	1.28	-.88
8. miscellaneous products, x_{OT}	.50	-.34

Table 3 indicates that except for dermatological agents, x_{DER}, and miscellaneous products, x_{OT}, the estimated income elasticities, $\hat{\eta}_i$, range

(14) Sato [16] calls $\sigma\eta_i$ "own-price" elasticities of demand.

from 1.18 to 2.01, i.e. their estimated income elasticities are above unity.(15) The estimated income elasticity of dermatological agents, $\hat{\eta}_{DER}$, is .48. This may be the result of changes in the living standard as the Japanese economy has expanded: for example, frost bite used to be a common phenomenon during winter, but as diet and heating facilities improved it has become rare except in some rural villages.

Table 4

Time Response Form of Distributed Lags in Industrial Demand Equations

time lags	x_{NS} $\{e^{-ak}\}$ a=2.0927	x_{CR} $\{k^{s-1}e^{-k}\}$ s=7.1389	x_{DU} $\{e^{-ak}\}$ a=.6635	x_V $\{e^{-ak}\}$ a=15.029	x_{NU} $\{e^{-ak}\}$ a=14.4689	x_{DER} $\{e^{-ak}\}$ a=13.0273	x_{BC} $\{e^{-ak}\}$ a=2.075	x_{OT} $\{k^{s-1}e^{-k}\}$ s=2.0757
0	.8766	.0004	.4850	1.0	1.0	1.0	.8744	.3814
1	.1082	.0102	.2498	0	0	0	.1098	.2957
2	.0134	.0452	.1287				.0139	.1683
3	.0016	.0973	.0663				.0017	.0844
4	.0002	.1408	.0341				.0002	.0395
5	0	.1586	.0176				0	.0177
6		.1503	.0091					.0077
7		.1255	.0047					.0033
sum of coefficients above	1.0	.7283	.9953	1.0	1.0	1.0	1.0	.9980

(15) One might wish to compare these estimated income elasticities of pharmacentical products with income elasticities of other consumer goods in Japan. The only source we have at hand is Tsujimura's work [20] reported in [17] . His income elasticity estimates for 1960 are for more aggregated expenditure categories. Among the 16 categories he studied, only five register income elasticities above unity, and they are cereals; rent; furniture and household equipment; recreation and entertainment; and education and micellaneous expenses. The estimated income elasticity of the last item, under which pharmaceutical products are classified, is 1.07.

On examining Table 4 to see whether the Koyck lags as indicated by (2-10) are valid or not, we find that in seven out of eight groups the Koyck lags indeed are appropriate.(16) The exception is group II, cardiovascular agents and agents affecting respiratory organs, whose time response is well distributed over long periods: the sum of the coefficients after 8 periods amounts to .7283. On the other hand, the following three products have a spontaneous time response: (i) vitamin preparations; (ii) nutrients, tonics, and alternatives, and (iii) dermatological agents.

To estimate the gamma distributed lags one needs to specify the length of lags m in $\{\sum_{k=1}^{m} k^{s-1}e^{-k}\}$. In this study we first used the maximum length of lags which were available from our data. When the estimated length of lags was shorter than the maximum length of lags, we cut the length shorter at the point where the lag coefficients were in the neighborhood of zero and we re-estimated the equation.(17) The maximum length of lags available from our data for the industrial demand equations was 15.

<u>4.2</u> <u>Market Share Equations</u>: The market share equation to be estimated was given by (2-13)'. Table 5 below presents the time response form of distributed lags:

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(16) In the case of miscellaneous products the coefficients decline as a Koyck type lag indicates. However, the Koyck lag formulation with $\{e^{-ak}\}$ yielded much poorer results than with $\{k^{s-1}e^{-k}\}$, judged by such rough measures as the estimated standard errors and the coefficient of determination. Hence, the latter form was retained.

(17) The strategy of choosing m in $\{k^{s-1}e^{-k}\}$ is discussed in [21] . There it is suggested that one may choose m such that the estimated variance of the equation is minimized. This strategy tends to be costly because it requires examination of the range of m, and furthermore the experiment in [21] indicates that the estimated coefficients do not vary much over the different values of m. Hence in this study we followed the method explained above. For (3-4), (3-5) and (3-6), however, we retained the lags up to the second period although the estimated Koyck type lag coefficients, a, were around 13.03 to 15.03,indicating instantaenous reactions.

Table 5

Time Response Form of Distributed Lags in Market Share Equations

time lags	E_{NS} $\{e^{-ak}\}$ a=2.0474	E_{CR} $\{e^{-ak}\}$ a= 7.8	E_{DU} $\{k^{s-1}e^{-k}\}$ s= 3.9282	E_V $\{e^{-ak}\}$ a= .7715	E_{NU} $\{k^{s-1}e^{-k}\}$ s=3.6219	E_{DER} $\{k^{s-1}e^{-k}\}$ s=3.4408
0	.8709	.9996	.0670	.5377	.0964	.1180
1	.1124	.0004	.1875	.2486	.2183	.2357
2	.0145	0	.2262	.1149	.2326	.2333
3	.0019		.1932	.0531	.1819	.1732
4	.0003		.1366	.0246	.1201	.1099
5	0		.0857	.0114	.0713	.0631
6			.0495	.0053	.0393	.0338
sum of coefficients above	1.0	1.0	.9457	.9956	.9599	.9670

Table 5 indicates that in three out of six cases the Koyck lag formulation holds, and especially, E_{CR}, has an almost instantaneous response function. In three cases, i.e., E_{DU}, E_{NU}, and E_{DER}, the peak of lags occurs around the second lagged period.

In formulating the market share equations, (2-11) and (2-12), we have assumed constant elasticity with respect to goodwill. Table 6 below presents the estimates of elasticities of Eisai products with respect to Eisai's goodwill. Three groups of products, i.e. vitamin preparations, E_V; nutrients, tonics and alternatives, E_{NU}; and dermatological agents, E_{DER}, give elasticity estimates well above unity. It may be interesting to note that these are the products in which Eisai has had relative strength compared to its competitors.

Table 6

Estimates of Elasticities of Eisai Products with Respect to Eisai's Goodwill

Product groups	Elasticity Estimates
1. agents affecting the nervous systems and sensory organs, E_{NS}	.77
2. cardiovascular agents and agents affecting respiratory organs, E_{CR}	.66
3. agents affecting digestive organs and agents for uro-genital and anal organs, E_{DU}	.44
4. vitamin preparations, E_V	1.47
5. nutrients, tonics and alternatives, E_{NU}	1.52
6. dermatological agents, E_{DER}	1.55

For the estimation of the market share equations, the nonlinear two-stage least squares method [22] was employed. The maximum length of lags available from our data was six. Consistency of the nonlinear least squares method was proven in [8] for cases in which the explanatory variables were nonstochastic and bounded. In our case the advertising expenditure and market share are simultaneously determined: at least the contemporaneous market share and advertising expenditure are mutually dependent, thus invalidating the assumption of nonstochastic explanatory variables. The nonlinear two-stage least squares (NL2SLS) is designed to 'purge' the explanatory variables which are correlated with the error term from the linearized part of the nonlinear equation in the same manner as the two-stage least squares method operates in the linear model. However, consistency of NL2SLS is not yet known, and thus one might argue that one should use the maximum likelihood (ML) estimation procedure such as [5] , since consistency of the ML estimator has been proven by Wald [24]. However, it is usually difficult to verify that all of the eight assumptions Wald makes indeed hold for a particular case. Furthermore, there is always the problem of small sample bias.(18) At best, NL2SLS estimates presented here should be treated as an experimental exercise.

4.3 Advertising Expenditure: The advertising expenditure equation was given by (2-32). Estimation of this equation presents the following two interesting empirical points: (i) time response form of distributed lags, and (ii) estimation of the depreciation rate of goodwill, δ. Recall that goodwill, AE_t, is given by

$$AE_t = \sum_{k=0}^{\ell} (1-\delta)^k \, PRE_{t-k}$$

where PRE_t is the advertising expenditure at time t. Due to the constraint of limited data we set ℓ at 7. Then the advertising expenditure equation, PRE_t, is given by

$$PRE_t = k_1 \tau(L) \, (V_t - V_{t-1}) + \sum_{k=0}^{7} (1-\delta)^k \, PRE_{t-k-1} + \varepsilon_t$$

where $\tau(L) = \sum_{k=0}^{7} k^{s-1} e^{-k} L^{k-1}$, and ε_t is the disturbance term. The non-linear estimation method (NL2SLS) allows us to estimate δ, which enters in the equation as the 8th order polynomial. The estimated $\hat{\delta}$ is given in (3-16) as .4494. This indicates that approximately 45 percent of goodwill will depreciate in one period (i.e. half a year), indicating a high rate of depreciation.

(18) Given these difficulties one way of examining different nonlinear estimation techniques is to conduct Monte Carlo experiments, although any Monte Carlo experiment suffers from its intrinsic problem that results depend on the particular specification of the model, nature of exogenous variables and error term. Goldfeld and Quant [7] examined some aspects of simple nonlinear simultaneous equations using the Monte Carlo method. As exogenous variables for NL2SLS we used the following: population, consumer expenditure, price index of consumer expenditure, competitors' advertising expenditure, and the general price index of all pharmaceutical products.

We used this estimate of the depreciation rate to derive goodwill of competitors, AO_t, and hence it is given as

$$AO_t = \sum_{k=0}^{7} (.5506)^k \, PRO_{t-k}$$

where PRO_t is the competitors' advertising expenditure at time t. To obtain some measure of real advertising expenditure, we deflated the current advertising expenditure by the price index of consumer services. This price index is chosen because it reflects the range of advertising activities of the pharmaceutical industry, i.e., from advertising in the mass media to expense account entertaining.

The time response form of lags in the advertising expenditure equation is presented in Table 7 below, indicating that the peak response occurs at the first lagged period.

Table 7

Time Response Form of Distributed Lags in Advertising Expenditure Equation

time lags	coefficients $\{k^{s-1}e^{-k}\}$ s=2.7009
0	.2411
1	.2883
2	.2114
3	.1269
4	.0682
5	.0342
6	.0164
Sum of coefficients above	.9865

In the model presented in Section III the prices of pharmaceutical products of the industry and those of the Eisai Company were treated as exogenous variables. This is due to the fact that in Japan the prices of ethical drugs are determined annually by the number of medicare points each product can get from the Medicare Commission. The prices of pharmaceutical products sold over the counter are influenced by the prices of their counterparts in ethical drugs. Hence without loss of validity in the model, the prices may be treated exogenously.(19)

V. COMPETITORS' REACTION TO THE FIRM'S ADVERTISING POLICY: A SIMULATION EXPERIMENT

The model given in Section III was solved by a modified Siedel method. First, a sample period simulation was made to check whether the model explains the actual values of the endogenous variables, and for this purpose the actual values of the endogenous variables were used. The computed values of endogenous variables were reasonably close to the actual values, judged by the Theil inequality coefficients which ranged in our case from .003 to .09. Since there are no well developed significant tests for the coefficients or for any other measure of sample period simulation within the framework of finite sample distributions, the Theil inequality coefficients given here should be regarded as a rough test of the workability of the model.

After the sample period simulation was done, we made two forecasting exercises for the four and half years between 1970.I and 1974.I to examine how a possible competitors' reaction to the Eisai Company's advertising policy would influence the various activities of the firm. In the first

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(19) This does not mean that there is no "price cutting." This effectively does happen, behind the scences and by personal agreement between the retail salesman and the physician or pharmacist, usually in the form of adding a bonus supply of drugs to the amount purchased at the official rate. The costs of bonus supplies are regarded by the firm as sales and promotion expenditures, rather than being interpreted as "price cutting" in the usual sense of price competition. Recently the Japanese government banned the practice of adding a bonus supply.

forecasting simulation we assumed that the competitors' advertising activity does not respond to that of the Eisai Company. In the second forecasting simulation, we relax this assumption by introducing a "reaction function" which expresses the competitors' advertising expenditure as a function of Eisai's advertising expenditure.

First, we start with a basic forecast based on the set of exogenous variables given in Table 8. In Table 8 most of the key exogenous variables such as population, consumption, and medicare expenditure are based on the time trend of the period from 1959.I to 1968.II. The assumptions which gave rise to the exogenous variables are given under Table 8. If any other reasonable forecasts of national economy are available, one may use them instead of the time trend values.

The time trend estimate of consumption expenditure, CE_t, gives an average growth rate of approximately 2.76 percent per half a year (or 5.6 percent per annum). The consumption expenditure has grown at an average rate of 4.1 percent per half a year (or 8.4 percent per annum) in the five years between October 1964 and September 1969. In view of this past performance of the Japanese economy, 2.76 percent per half a year may be a conservative estimate, if one supposes that the high growth rate of the late sixties is to be continued.

From the exogenous variables we have obtained the forecast values of the endogenous variables. Forecasts of industrial demand for pharmaceutical products are given in Table 9. Table 10 presents forecasts of the Eisai Company's activities and its market shares.

Table 10 indicates that given the exogenous variables of Table 8, the Eisai Company will double the value of net sales in four years from 170.64 billion yen in 1970.I to 371.5 billion yen in 1974.I, indicating a rate of growth of 117.7 percent. As indicated in Table 1 in Section II, the Eisai Company grew 111 percent in the four years between 1964 and 1968. Table 10 indicates that the company can expect to grow in similar fashion in the coming four years.

Table 8

A Set of Exogenous Variables Used for the Forecasting Exercises

	1970.I	1970.II	1971.I	1971.II	1972.I	1972.II	1973.I	1973.II	1974.I
N	103488	104006	104524	105041	105559	106077	106594	107112	107630
CE	27076	27891	28706	29521	30336	31151	31966	32781	33596
MED	1192	1247	1302	1357	1412	1467	1522	1577	1632
PRO	45086	46529	48018	49554	51140	52776	54465	56208	58007

Assumptions

1. Population, N, grows according to the time trend of 1959.I--1968.II.
2. Consumer expenditure in billions of 1965 yen, CE, grows according to the time trend of 1959.I--1968.II.
3. Medicare expenditure, MED, grows according to the time trend of 1959.I-1968.II.
4. Competitors' advertising expenditure, PRO, grows at 3.2 percent per half a year (average of 1959.I--1968.II growth rates).
5. The relative prices, P_i/P_d, in the industrial demand equations (3-1)--(3-8) stay at the same level as in 1969.II.
6. The prices of Eisai Products, PE_i, stay at the same level as in 1969.II
7. The rate of growth of chemotherapeutics and antibiotics of the Eisai Company is the same as that of the industry.
8. The sale of miscellaneous products of the Eisai Company stays at the same level as in 1969.II.

However, can the company expect to maintain such a high rate of growth ? In the past the company was sufficiently small and thus competitors have not been sensitive to the expansion of the Eisai Company. If the company maintains its high rate of growth by cutting into the competitors' market shares, the latter will feel it necessary to react to Eisai's sales activities.

The fact that in the past Eisai's competitors did not react much to its advertising activity may be indicated in the following regression over the

Table 9

Forecasts of Industrial Demand Based on the Set of Exogenous Variables Given in Table 8

	1970.I	1970.II	1971.I	1971.II	1972.I	1972.II	1973.I	1973.II	1974.I
x_{NS}	1.0649	1.1123	1.1626	1.2138	1.2658	1.3185	1.3719	1.4261	1.4809
x_{CR}	.6023	.6829	.7749	.8755	.9799	1.0838	1.1846	1.2814	1.3749
x_{DU}	.4737	.4997	.5239	.5473	.5703	.5932	.6162	.6393	.6626
x_{V}	.7159	.7420	.7683	.7946	.8212	.8478	.8746	.9015	.9285
x_{NU}	.7622	.8018	.8421	.8833	.9252	.9679	1.0114	1.0556	1.1006
x_{DER}	.2024	.2054	.2083	.2112	.2140	.2169	.2196	.2223	.2250
x_{BC}	.8074	.8379	.8699	.9024	.9351	.9681	1.0013	1.0348	1.0686
x_{OT}	.1544	.1576	.1603	.1629	.1653	.1676	.1700	.1722	.1745
X_{NS}	110206	115690	121515	127498	133615	139863	146240	152750	159389
X_{CR}	62329	71023	80999	91962	103438	114968	126271	137256	147980
X_{DU}	49019	51971	54764	57489	60201	62927	65683	68479	71317
X_{V}	74089	77175	80302	83471	86681	89934	93227	96562	99938
X_{NU}	78883	83391	88024	92781	97664	102671	107805	113066	118455
X_{DER}	20947	21362	21774	22185	22595	23003	23410	23816	24221
X_{BC}	83558	87146	90927	94786	98708	102694	106738	110843	115008
X_{OT}	15975	16388	16757	17107	17447	17783	18117	18449	18779
X	495006	524144	555062	587277	620349	653843	687489	721220	755087

Table 10

Forecasts of Eisai Activities

	1970.I	1970.II	1971.I	1971.II	1972.I	1972.II	1973.I	1973.II	1974.I
E_{NS}	4086	4359	4675	5041	5435	5865	6332	6841	7396
E_{CR}	3531	3942	4420	4968	5551	6163	6796	7453	8140
E_{DU}	2480	2593	2708	2826	2946	3070	3197	3330	3469
E_{V}	3965	4435	4983	5643	6408	7297	8324	9513	10891
E_{NU}	1943	2277	2640	3042	3505	4048	4678	5430	6315
E_{DER}	796	852	914	990	1083	1192	1318	1464	1631
PRE	4864	5252	5674	6165	6706	7311	7980	8728	9566
V	17064	18641	20438	22537	24848	27431	30302	33526	37151
AE	7518	8139	8798	9553	10379	11300	12322	13460	14734
Market Shares									
E_{NS}	.0371	.0377	.0385	.0395	.0407	.0419	.0433	.0448	.0464
E_{CR}	.0566	.0555	.0546	.0540	.0537	.0536	.0538	.0543	.0550
E_{DU}	.0506	.0499	.0495	.0492	.0489	.0488	.0487	.0486	.0486
E_{V}	.0535	.0575	.0621	.0676	.0739	.0811	.0893	.0985	.1090
E_{NU}	.0246	.0273	.0300	.0328	.0359	.0394	.0434	.0480	.0533
E_{DER}	.0380	.0399	.0420	.0446	.0479	.0518	.0563	.0615	.0674

sample period, 1959.II -- 1969.II.

$$\text{(5-1)} \qquad \Delta PRO_t = \underset{(5.0017)}{7.2746} \frac{1}{G} \sum_{k=1}^{7} e^{-ak} \Delta PRE_{t-k+1} + \underset{(981.3894)}{327.0957}$$

$$\underset{(1.8401)}{a=1.4952} \qquad \bar{R}^2 = .292 \quad DW = 1.18 \quad \text{NLLS}$$

where ΔPRO_t = increment in the competitors' advertising expenditure at time t

ΔPRE_t = increment in Eisai's advertising expenditure at time t.

The estimated coefficients and the coefficient of determination adjusted for degrees of freedom are hardly significant.

However, the competitors may now tend to react strongly against Eisai's advertising activity. As an exercise, suppose that from 1970.I the competitors' advertising activity begins to react as given in (5-1). Then what will happen to Eisai's net sales and market shares ? Table 11 presents the result.

From Table 11 we notice that Eisai's net sales, V, will increase by 58.9 percent in four years from 168.1 billion yen in 1970.I to 267.0 billion yen in 1974.I, and this is a marked decline from the rate of growth of 117.7 percent as shown in Table 10. If the Eisai Company is to maintain its past high rate of growth in the face of the competitors' reaction, the company will have to increase its advertising activities. Also it will have to emphasize efficient research and development (R&D) expenditure which may yield new products.

Table 11

Forecasts of Eisai Activities When the Competitors' Advertising Expenditure Reacts to that of Eisai as Given in (5-1)

	1970.I	1970.II	1971.I	1971.II	1972.I	1972.II	1973.I	1973.II	1974.I
E_{NS}	3980	4129	4322	4507	4709	4906	5110	5317	5527
E_{CR}	3441	3750	4122	4496	4893	5268	5634	5983	6318
E_{DU}	2477	2577	2670	2754	2832	2905	2974	3042	3109
E_V	3844	4110	4417	4725	5066	5413	5777	6159	6558
E_{NU}	1931	2215	2491	2750	3019	3296	3584	3903	4243
E_{DER}	790	826	857	887	922	957	994	1034	1074
PRE	4838	5109	5415	5683	5982	6261	6543	6832	7126
V	16807	17831	19109	20240	21551	22769	24046	25360	26699
AE	7498	7965	8489	8924	9413	9863	10322	10788	11262
Market Shares									
E_{NS}	.0361	.0357	.0356	.0353	.0352	.0351	.0349	.0348	.0347
E_{CR}	.0552	.0528	.0509	.0489	.0473	.0458	.0446	.0436	.0427
E_{DU}	.0505	.0496	.0488	.0479	.0470	.0462	.0453	.0444	.0436
E_V	.0519	.0533	.0550	.0566	.0584	.0602	.0620	.0638	.0656
E_{NU}	.0245	.0266	.0283	.0296	.0309	.0321	.0332	.0345	.0358
E_{DER}	.0377	.0387	.0394	.0400	.0408	.0416	.0425	.0434	.0443

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